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Stabilization of Discrete Nonlinear Singularly Perturbed System with Time-Delay Represented by a Coupled Multimodel

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***Abstract*—This paper investigates the problem of state feedback stabilisation for discrete nonlinear singularly perturbed system with time-delay which is represented by a coupled multimodel. Based on an appropriate Lyapunov function, new sufficient conditions are given as a set of Linear Matrix Inequalities (LMIs) that are used to get the gains of controllers. Numerical example is given to illustrate the effectiveness of the proposed method.**

I. INTRODUCTION

In the past three decades, singularly perturbed system (SPS) have been intensively studied due to their ability to represent many industriel systems which are characterized by slow and fast dynamics and in the majority of cases these systems are also non-linear. This increases their complexity. Subsequently, traditional methods of analysis and synthesis become inefficient. In recent years, many techniques of simplification have been developed to reduce the complexity of modeling, control and analysis [4]. In most studies, the reduction of this complexity leads to a lack of information. This reduction is achieved either by neglecting certain phenomena (chemical reaction for example), or by elimination of certain parameters which do not have a significant importance on the dynamics of the system [[5], [4], [6], [7]]. Therefore, it is necessary to use another powerful technique to confront these problems. In this context, we will focus on the multimodel approach that is known by its power to handle with complex systems. In deed, this approach represents a powerful tool for modeling, observation and control of complex systems [8]. The basic idea of this approach is to decompose the global problem into a set of sub-problems that are simpler and easier to solve.
On the other hand, time-delay is commonly encountered in several industriel system. It can lead to instability or can degrade the performance of systems. Thus, it is important to design a controller that stabilize the closed loop system. Several research have studied the control problem for nonlinear singularly perturbed system [[9], [10], [11], [13], [12]]. In [[9], [10]], controller design for continuous-time nonlinear SPS with time-delay has been treated. However, these results can not handel with discrete-time cases. In [13], the problem of state feedback stabilization for fuzzy SPSs is treated. But, time-delay wasn't considered in this work. Chen et al. were investigated a control problem for discrete-time time-delay fuzzy SPS was described by fuzzy "IF-THEN" rules [11]. Liu et al. were considered the controller design for discrete-time fuzzy SPS under the fast-time version [15]. But this class of system does not always conserve the time scale character [14]. This paper present a new result on state feedback stabilization for discrete-time time-delay nonlinear SPS with slow rate which known by its preservation of time scale character. First, the discrete nonlinear SPS is represented by a discrete coupled multimodel by using Convex Polytopic Transformation (CPT). After that, based on an appropriate Lyapunov function, a new method for designing a controller is presented.
This paper is organized as follow. In section 2, nonlinear SPS with time-delay (NSPSD) and its representation with a coupled multimodel are represented. The main results are given in section 3, where the state feedback stabilization problem is treated. Simulation results for a discrete NSPSD show the effectiveness of the presented method in section 4. In section 5, a conclusion finishes this paper.

II. COUPLED MULTIMODEL REPRESENTATION OF DISCRETE NONLINEAR SINGULARLY PERTURBED SYSTEM WITH TIME-DELAY

Consider a discrete non linear singularly perturbed system (NLSPS) with time-delay:

$$\begin{aligned} x_1(k+1) &= f_{NL}(x_1(k), x_2(k), x_1(k-d), x_2(k-d), u(k)) \\ x_2(k+1) &= \varepsilon g_{NL}(x_1(k), x_2(k), x_1(k-d), x_2(k-d), u(k)) \end{aligned} \tag{1}$$

where $x_1 \in R^n$, $x_2 \in R^m$ are system states and $u(k)$ is control input.
d: positif integer which is the time-delay.
ε: is a small positive parameter.
$f_{NL} : R^n \times R^m \to R^n$ and $g_{NL} : R^n \times R^m \to R^m$.
The discrete NLSPS with time-delay (1) can be represented by a coupled multimodel using Convex Polytopic Transformation (CPT). This method does not present an error of approximation and the choice of decision variable is realized in a systematic way [16]. It assumes that all non constant terms are bounded. If r is the number of nonlinearity distincts existent in nonlinear

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system (1), then the obtained multimodel is composed of $2^r = N_m$ sub-models.
The following Lemma will be used to manipulate the nonlinear term.

Lemma 1 [16]

Let $H(x(t), u(t))$ a continuous and bounded function on the domain $D \subset \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$,
with $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$.
Then there are two functions G_i $(i = 1, 2)$

$$\begin{array}{l} G_i : D \mapsto [0, 1] \\ (x(t), u(t)) \mapsto G_i(x(t), u(t)) \end{array} \quad (2)$$

$G_1(x(t), u(t)) + G_2(x(t), u(t)) = 1$, with:

$$H(x(t), u(t)) = G_1(x(t), u(t)).H_1 + G_2(x(t), u(t)).H_2$$

For all $H_1 \geq maximum_{x,u \in D} H(x(t), u(t))$, $H_2 \leq minimum_{x,u \in D} H(x(t), u(t))$
The functions G_1 and G_2 are given as follow:

$$G_1(x(t), u(t)) = \frac{H(x(t), u(t)) - H_2}{H_1 - H_2} \quad (3)$$

$$G_2(x(t), u(t)) = \frac{H_1 - H(x(t), u(t))}{H_1 - H_2} \quad (4)$$

In particular: $H_1 = maximum_{x,u \in D} H(x(t), u(t))$ and $H_2 = minimum_{x,u \in D} H(x(t), u(t))$
The CPT can be summarized by the following steps.

Step 1: Write the system Eqn. (1) as a quasi linear parameter varying (quasi-LPV) form:

$$x(k+1) = A_\varepsilon(x_1(k), x_2(k))x(k) + D_\varepsilon(x_1(k), x_2(k))x(k-d) \quad (5)$$

where $A_\varepsilon(x_1(k), x_2(k)) \in \mathbb{R}^{n \times n}$ and $D_\varepsilon(x_1(k), x_2(k)) \in \mathbb{R}^{n \times n}$.
r_1 is the number of nonlinear terms in $A_\varepsilon(x_1(k), x_2(k))$.
r_2 is the number of nonlinear terms in $D_\varepsilon(x_1(k), x_2(k))$.
$r = r_1 + r_2$

Step 2: Set up the bound of each nonlinear term in $A_\varepsilon(x_1(k), x_2(k))$ and $D_\varepsilon(x_1(k), x_2(k))$.

Step 3: Let
$H_j(x_1(k), x_2(k))$, $j = 1, ..., r$ are the nonlinear term in $A_\varepsilon(x_1(k), x_2(k))$ and $D_\varepsilon(x_1(k), x_2(k))$.
Each nonlinear term $H_j(x_1(k), x_2(k))$, $j = 1, ..., r$ can be written according to its $G_1(x_1(k), x_2(k))$, its $G_2(x_1(k), x_2(k))$, its maximum (H_1) and its munimum (H_2) according to lemma 1.
To each sub-model i corresponds a p-uplet σ_i that codes the partitions of the decision variables existing in the corresponding weighting function.
The weighting function $\mu_i(x_1(k), x_2(k))$, $i = 1, .., N_m$ is obtained by multiplying the function $G_{j,\sigma_i^j}(x_1(k), x_2(k))$ that describe the partitions of the decision variable:

$$\mu_i(x_1(k), x_2(k)) = \prod_{j=1}^{r} G_{j,\sigma_i^j}(x_1(k), x_2(k)) \quad (6)$$

where σ_i^j is the index in the j^{th} position in the p-uplet σ_i.
$A_\varepsilon(x_1(k), x_2(k))$ can be written as a linear combination of constant matrix $A_j(\varepsilon)$:

$$A_\varepsilon(x_1(k), x_2(k)) = A_0(\varepsilon) + \sum_{j \in E_A} H_j(x_1(k), x_2(k))A_j(\varepsilon) \quad (7)$$

where E_A includes all the indexes corresponding to the premises variables that exist in $A_\varepsilon(x_1(k), x_2(k))$.
$A_0(\varepsilon)$ and $A_j(\varepsilon)$ have the same dimension of the matrix $A_\varepsilon(x_1(k), x_2(k))$.
All the constant element of $A_\varepsilon(x_1(k), x_2(k))$ are found in $A_0(\varepsilon)$.
For the matrix $A_j(\varepsilon)$, at the position corresponding to $H_j(x_1(k), x_2(k))$, the constant element is equal to 1 and the other remaining positions are equal to zero.
Using the equation (7), the matrix $A_i(\varepsilon)$, $i = 1, ...N_m$ is given as follow:

$$A_i(\varepsilon) = A_0(\varepsilon) + \sum_{j \in E_A} z_{j,\sigma_i^j} A_j(\varepsilon) \quad (8)$$

$D_\varepsilon(x_1(k), x_2(k))$ can be found in the same way.

Step 4: Write the system in the form of multimodel and calculate the weighting functions $\mu_i(x)$, $i = 1...N_m$.
The coupled multimodel is given as follow:

$$\begin{array}{c} x(k+1) = \sum_{i=1}^{N_m} \mu_i(\xi(k))A_i(\varepsilon)x(k) + \sum_{i=1}^{N_m} \mu_i(\xi(k))D_i(\varepsilon) \\ x(k-d) + \sum_{i=1}^{N_m} \mu_i(\xi(k))B_i(\varepsilon)u(k) \end{array} \quad (9)$$

where

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \quad B_i(\varepsilon) = \begin{bmatrix} B_{i1} \\ \varepsilon B_{i2} \end{bmatrix}$$

$$A_i(\varepsilon) = \begin{bmatrix} A_{i11} & A_{i12} \\ \varepsilon A_{i21} & \varepsilon A_{i22} \end{bmatrix}, \ D_i(\varepsilon) = \begin{bmatrix} D_{i11} & D_{i12} \\ \varepsilon D_{i21} & \varepsilon D_{i22} \end{bmatrix}$$

N_m is the number of sub-models, $i = 1, 2, ...N_m$.

A_{i11}, A_{i12}, A_{i21}, A_{22}, D_{i11}, D_{i12}, D_{i21}, D_{i22}, B_{i1} and B_{i2} are appropriate dimension matrices.
$\mu_i(\xi(k))$ are the weighting functions that ensure the transition between the sub-models. They are characterized by the following properties:

$$\sum_{i=1}^{N_m} \mu_i(\xi(k)) = 1, \quad \forall k \quad (10)$$

$$0 \leq \mu_i(\xi(k)) \leq 1, \quad \forall k, \quad i = 1, 2, ...N_m \quad (11)$$

$\xi(k)$ is the decision variable. It can be signal input, signal output or system state.
The system (9) can be written as:

$$x(k+1) = \bar{A}(\varepsilon)x(k) + \bar{D}(\varepsilon)x(k-d) + \bar{B}(\varepsilon)u(k) \quad (12)$$

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where

$$\bar{A}(\varepsilon)=\sum_{i=1}^{N_m}\mu_i(\xi(k))A_i(\varepsilon),\ \bar{D}(\varepsilon)=\sum_{i=1}^{N_m}\mu_i(\xi(k))D_i(\varepsilon)$$

$$\bar{B}(\varepsilon)=\sum_{i=1}^{N_m}\mu_i(\xi(k))B_i(\varepsilon) \quad (13)$$

In the following, we propose to design a multimodel control law , which is given as follow:

$$u(k)=\bar{K}x(k),\bar{K}=\sum_{i=1}^{N_m}K_i \quad (14)$$

Then the resulting closed-loop system is given by:

$$x(k+1)=\bar{A}_{BF}(\varepsilon)x(k)+\bar{D}(\varepsilon)x(k-d) \quad (15)$$

where

$$\bar{A}_{BF}(\varepsilon)=\bar{A}(\varepsilon)+\bar{B}(\varepsilon)\bar{K}$$

We recall the following lemmas, in order to establish our main results.

Lemma 2 [1]:

For any positive scalar ε^*, if the following conditions are verified:

$$F_1\geq 0,$$

$$\varepsilon^{*2}F_1+\varepsilon^*F_2+F_3<0,$$

$$F_3<0,$$

then we get

$$\varepsilon^2F_1+\varepsilon F_2+F_3<0,\quad for\quad \varepsilon\in[0,\varepsilon^*]$$

Lemma 3 [2]:

Let $V\in\mathbb{R}^{n_1}$, $W\in\mathbb{R}^{n_2}$ and $N\in\mathbb{R}^{n_1*n_2}$.
Then for any matrices $M\in\mathbb{R}^{n_1*n_1}$, $S\in\mathbb{R}^{n_1*n_2}$ and $Z\in\mathbb{R}^{n_2*n_2}$ satisfying: $\begin{bmatrix} M & S\\ S^T & Z\end{bmatrix}\geq 0$, we have,

$$-2V^TNW\leq\begin{bmatrix} V\\ W\end{bmatrix}^T\begin{bmatrix} M & S-N\\ S^T-N^T & Z\end{bmatrix}\begin{bmatrix} V\\ W\end{bmatrix} \quad (16)$$

III. Stabilisation of Nonlinear Singularly Perturbed System with Time-delay represented by a Coupled Multimodel

Theorem 1

Consider the system (9).
Let the positif real $\varepsilon^*>0$ and $d>0$ a positive integer, if there exist symmetric matrices define positif $X>0$, $\tilde{Q}>0$ and $\tilde{M}$, $\tilde{R}$ a symmetric matrix and matrices $\tilde{W}$, $\tilde{K}$ satisfying the following LMIs:

$$\tilde{\Omega}'_{ii}(\varepsilon^*)<0 \quad (17)$$

$$\tilde{\Omega}'_{ii}(0)<0 \quad (18)$$

$$\tilde{\Omega}'_{ij}(\varepsilon^*)+\tilde{\Omega}'_{ji}(\varepsilon^*)<0 \quad (19)$$

$$\tilde{\Omega}'_{ij}(0)+\tilde{\Omega}'_{ji}(0)<0 \quad (20)$$

$$\begin{bmatrix}\tilde{R} & \tilde{W}\\ \tilde{W}^T & \tilde{M}\end{bmatrix}\geq 0 \quad (21)$$

where

$$\tilde{\Omega}'_{ij}(\varepsilon)=\begin{bmatrix}(1,1) & * & * & *\\ (2,1) & (2,2) & * & *\\ (3,1) & (3,2) & (3,3) & *\\ (4,1) & (4,2) & (4,3) & (4,4)\end{bmatrix} \quad (22)$$

with
$(1,1)=-X+d\tilde{R}+\tilde{W}^T+\tilde{W}+\tilde{Q}$
$(2,1)=-\tilde{W}^T$
$(2,2)=-\tilde{Q}$
$(3,1)=A_i(\varepsilon)X+B_i(\varepsilon)\tilde{K}_j$
$(3,2)=D_i(\varepsilon)X$
$(3,3)=-X$
$(4,1)=\sqrt{d}A_i(\varepsilon)X+\sqrt{d}B_i(\varepsilon)\tilde{K}_j-\sqrt{d}X$
$(4,2)=\sqrt{d}D_i(\varepsilon)X$
$(4,3)=0$
$(4,4)=\tilde{M}-2X$

Then there exists a controller given by equation (23) such that the closed loop system is asymptotically stable $\forall\varepsilon\in[0,\varepsilon^*]$.

$$u(k)=\sum_{i=1}^{N_m}K_ix(k),\ avec,\ K_i=\tilde{K}_iX^{-1} \quad (23)$$

Proof

We consider the following Lyapunov-Krasovskii functional:

$$V(k)=V_1(k)+V_2(k)+V_3(k) \quad (24)$$

where
$V_1(k)=x^T(k)Px(k)$
$V_2(k)=\sum_{i=k-d}^{k-1}x^T(i)Qx(i)$
$V_3(k)=d\sum_{\theta=-d}^{(-1)}\sum_{i=k+\theta}^{k-1}\eta^T(i)M\eta(i),\quad \eta(i)=x(i+1)-x(i)$
with P, Q and M are symmetric and positive definite matrices.

$$Since\quad \eta(i)=x(i+1)-x(i)$$

then we have:

$$x(k-d)=x(k)-\sum_{i=k-d}^{k-1}\eta(i) \quad (25)$$

Substituting (25) into (12), we obtain:

$$x(k+1)=A_{dd}(\varepsilon)x(k)-D(\varepsilon)\sum_{i=k-d}^{k-1}\eta(i) \quad (26)$$

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with $A_{dd}(\varepsilon) = \bar{A}(\varepsilon) + \bar{B}(\varepsilon)\bar{K} + \bar{D}(\varepsilon)$

$$\begin{aligned}\Delta V_1(k) &= V_1(k+1) - V_1(k) \\ &= x^T[\bar{A}_{dd}^T(\varepsilon)P\bar{A}_{dd}(\varepsilon) - P]x(k) - 2\sum_{i=k-d}^{k-1} x^T(k)\bar{A}_{dd}^T(\varepsilon)P\bar{D}(\varepsilon) \\ &\quad \eta(i) + N^T(k)\bar{D}^T(\varepsilon)P\bar{D}(\varepsilon)N(k)\end{aligned} \quad (27)$$

where $N(k) = \sum_{i=k-d}^{k-1} \eta^T(i)$
Posing $\theta = \bar{A}_{dd}^T(\varepsilon)P\bar{D}(\varepsilon)$ and using Lemma 3, we obtain:

$$\begin{aligned}\Delta V_1(k) &\le x^T[\bar{A}_{dd}^T(\varepsilon)P\bar{A}_{dd}(\varepsilon) - P + dR + W^T - \theta^T + W - \theta \\ &+ \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon)]x(k) + x^T(k)[-W + \theta - \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon)]x(k-d) \\ &+ x^T(k-d)[-W^T + \theta^T - \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon)]x(k) + x^T(k-d) \\ &[\bar{D}^T(\varepsilon)P\bar{D}(\varepsilon)]x(k-d) + \sum_{i=k-d}^{k-1} \eta^T(i)M\eta(i)\end{aligned} \quad (28)$$

with R, W and M satisfying the following condition:

$$\begin{bmatrix} R & W \\ W^T & M \end{bmatrix} \ge 0 \quad (29)$$

$\Delta V_2(k)$ is given as:

$$\Delta V_2(k) = x^T(k)Qx(k) - x^T(k-d)Qx(k-d) \quad (30)$$

$\Delta V_3(k)$ can be written as follow:

$$\begin{aligned}\Delta V_3(k) &= dx^T(k)\bar{A}_{BF}^T(\varepsilon)M\bar{A}_{BF}(\varepsilon)x(k) + dx^T(k)\bar{A}_{BF}^T(\varepsilon) \\ &M\bar{D}(\varepsilon)x(k-d) + dx^T(k-d)\bar{D}^T(\varepsilon)M\bar{A}_{BF}(\varepsilon)x(k) \\ &+ dx^T(k-d)\bar{D}^T(\varepsilon)M\bar{D}(\varepsilon)x(k-d) \\ &- \sum_{i=k-d}^{k-1} \eta^(i)M\eta(i)\end{aligned} \quad (31)$$

with $\bar{A}_{BF}(\varepsilon) = \bar{A}(\varepsilon) + \bar{B}(\varepsilon)\bar{K} - I$
$\Delta V(k)$ is given as follow:

$$\begin{aligned}\Delta V(k) &= \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \\ &\le x^T(k)[\bar{A}_{dd}^T(\varepsilon)P\bar{A}_{dd}(\varepsilon) - P + dR + W^T - \theta^T + W - \theta \\ &- \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon) + Q + d\bar{A}_{BF}^T(\varepsilon)M\bar{A}_{BF}(\varepsilon)]x(k) + x^T(k)[-W \\ &+ \theta - \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon) + d\bar{A}_{BF}^T(\varepsilon)M\bar{D}(\varepsilon)]x(k-d) + x^T(k-d) \\ &[-W^T + \theta^T - \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon) + d\bar{D}^T(\varepsilon)M\bar{A}_{BF}(\varepsilon)]x(k) \\ &+ x^T(k-d)[\bar{D}^T(\varepsilon)P\bar{D}(\varepsilon) - Q + d\bar{D}^T(\varepsilon)M\bar{D}(\varepsilon)]x(k-d) \\ &\le \xi^T(k)\Omega(\varepsilon)\xi(k)\end{aligned} \quad (32)$$

where

$$\xi(k)^T = \begin{bmatrix} x(k) \\ x(k-d) \end{bmatrix}^T$$

$$\Omega(\varepsilon) = \begin{bmatrix} \Omega_{11}(\varepsilon) & * \\ \Omega_{21}(\varepsilon) & \Omega_{22}(\varepsilon) \end{bmatrix}$$

with

$$\begin{aligned}\Omega_{11}(\varepsilon) &= [\bar{A}^T(\varepsilon) + \bar{K}^T\bar{B}^T(\varepsilon)]P[\bar{A}(\varepsilon) + \bar{B}(\varepsilon)\bar{K}] - P + dR \\ &+ W^T + W + Q + d\bar{A}_{BF}^T(\varepsilon)M\bar{A}_{BF}(\varepsilon)]\end{aligned}$$

$$\Omega_{21}(\varepsilon) = -W^T + \bar{D}^T(\varepsilon)P[\bar{A}(\varepsilon) + \bar{B}(\varepsilon)\bar{K}] + d\bar{D}^T(\varepsilon)M\bar{A}_{BF}(\varepsilon)$$

$$\Omega_{22}(\varepsilon) = \bar{D}^T(\varepsilon)P\bar{D}(\varepsilon) - Q + d\bar{D}^T(\varepsilon)M\bar{D}(\varepsilon)$$

$\Delta V(k) < 0,\ \forall\varepsilon \in [0, \varepsilon^*]$ if and only if:

$$\Omega(\varepsilon) < 0,\ \forall\varepsilon \in [0, \varepsilon^*] \quad (33)$$

Applying the Schur Complement twice on (33), we get:

$$\Omega'(\varepsilon) = \begin{bmatrix} \Omega'_{11}(\varepsilon) & * & * & * \\ \Omega'_{21}(\varepsilon) & \Omega'_{22}(\varepsilon) & * & * \\ \Omega'_{31}(\varepsilon) & \Omega'_{32}(\varepsilon) & \Omega'_{33}(\varepsilon) & * \\ \Omega'_{41}(\varepsilon) & \Omega'_{42}(\varepsilon) & \Omega'_{43}(\varepsilon) & \Omega'_{44}(\varepsilon) \end{bmatrix} < 0 \quad (34)$$

with
$\Omega'_{11}(\varepsilon) = -P + dR + W^T + W + Q$
$\Omega'_{21}(\varepsilon) = -W^T$
$\Omega'_{22}(\varepsilon) = -Q$
$\Omega'_{31}(\varepsilon) = \bar{A}(\varepsilon) + \bar{B}(\varepsilon)\bar{K}$
$\Omega'_{32}(\varepsilon) = \bar{D}(\varepsilon)$
$\Omega'_{33}(\varepsilon) = -P^{-1}$
$\Omega'_{41}(\varepsilon) = \sqrt{d}\bar{A}(\varepsilon) + \sqrt{d}\bar{B}(\varepsilon)\bar{K} - \sqrt{d}I$
$\Omega'_{42}(\varepsilon) = \sqrt{d}\bar{D}(\varepsilon)$
$\Omega'_{43}(\varepsilon) = 0$
$\Omega'_{44}(\varepsilon) = -M^{-1}$
Multiplying equation (34) on the left and right by $diag = \{P^{-1}, P^{-1}, I, I\}$ and posing the following equalities:
$X = P^{-1}$, $\tilde{R} = P^{-1}RP^{-1}$, $\tilde{W} = P^{-1}WP^{-1}$, $\tilde{Q} = P^{-1}QP^{-1}$, $\tilde{K} = \bar{K}P^{-1}$
we get:

$$\tilde{\Omega}(\varepsilon) = \begin{bmatrix} \tilde{\Omega}_{11}(\varepsilon) & * & * & * \\ \tilde{\Omega}_{21}(\varepsilon) & \tilde{\Omega}_{22}(\varepsilon) & * & * \\ \tilde{\Omega}_{31}(\varepsilon) & \tilde{\Omega}_{32}(\varepsilon) & \tilde{\Omega}_{33}(\varepsilon) & * \\ \tilde{\Omega}_{41}(\varepsilon) & \tilde{\Omega}_{42}(\varepsilon) & \tilde{\Omega}_{43}(\varepsilon) & \tilde{\Omega}_{44}(\varepsilon) \end{bmatrix} < 0 \quad (35)$$

where
$\tilde{\Omega}_{11}(\varepsilon) = -X + d\tilde{R} + \tilde{W}^T + \tilde{W} + \tilde{Q}$
$\tilde{\Omega}_{21}(\varepsilon) = -\tilde{W}^T$
$\tilde{\Omega}_{22}(\varepsilon) = -\tilde{Q}$
$\tilde{\Omega}_{31}(\varepsilon) = \bar{A}(\varepsilon)X + \bar{B}(\varepsilon)\tilde{K}$
$\tilde{\Omega}_{32}(\varepsilon) = \bar{D}(\varepsilon)X$
$\tilde{\Omega}_{33}(\varepsilon) = -X$
$\tilde{\Omega}_{41}(\varepsilon) = \sqrt{d}\bar{A}(\varepsilon)X + \sqrt{d}\bar{B}(\varepsilon)\tilde{K} - \sqrt{d}X$
$\tilde{\Omega}_{42}(\varepsilon) = \sqrt{d}\bar{D}(\varepsilon)X$
$\tilde{\Omega}_{43}(\varepsilon) = 0$
$\tilde{\Omega}_{44}(\varepsilon) = -M^{-1}$
Since M is a symmetric positive-definite matrix, we have:

$$(X - M^{-1})M(X - M^{-1}) > 0 \quad (36)$$

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Inequality (36) can be rewritten as:

$$XMX - 2X > -M^{-1} \quad (37)$$

According to (37), we know that the following inequality is sufficient to inequality (35):

$$\tilde{\Omega}^{'}(\varepsilon) = \begin{bmatrix} \tilde{\Omega}^{'}_{11}(\varepsilon) & * & * & * \\ \tilde{\Omega}^{'}_{21}(\varepsilon) & \tilde{\Omega}^{'}_{22}(\varepsilon) & * & * \\ \tilde{\Omega}^{'}_{31}(\varepsilon) & \tilde{\Omega}^{'}_{32}(\varepsilon) & \tilde{\Omega}^{'}_{33}(\varepsilon) & * \\ \tilde{\Omega}^{'}_{41}(\varepsilon) & \tilde{\Omega}^{'}_{42}(\varepsilon) & \tilde{\Omega}^{'}_{43}(\varepsilon) & \tilde{\Omega}^{'}_{44}(\varepsilon) \end{bmatrix} < 0 \quad (38)$$

where
$\tilde{\Omega}^{'}_{11}(\varepsilon) = -X + d\tilde{R} + \tilde{W}^T + \tilde{W} + \tilde{Q}$
$\tilde{\Omega}^{'}_{21}(\varepsilon) = -\tilde{W}^T$
$\tilde{\Omega}^{'}_{22}(\varepsilon) = -\tilde{Q}$
$\tilde{\Omega}^{'}_{31}(\varepsilon) = \bar{A}(\varepsilon)X + \bar{B}(\varepsilon)\tilde{K}$
$\tilde{\Omega}^{'}_{32}(\varepsilon) = \bar{D}(\varepsilon)X$
$\tilde{\Omega}^{'}_{33}(\varepsilon) = -X$
$\tilde{\Omega}^{'}_{41}(\varepsilon) = \sqrt{d}\bar{A}(\varepsilon)X + \sqrt{d}\bar{B}(\varepsilon)\tilde{K} - \sqrt{d}X$
$\tilde{\Omega}^{'}_{42}(\varepsilon) = \sqrt{d}\bar{D}(\varepsilon)X$
$\tilde{\Omega}^{'}_{43}(\varepsilon) = 0$
$\tilde{\Omega}^{'}_{44}(\varepsilon) = \tilde{M} - 2X$

with $\tilde{M} = XMX$.

Substituting (13) into (38), we get

$$\tilde{\Omega}^{'}(\varepsilon) = \sum_{i=1}^{Nm} \mu_i^2(\xi(k))\tilde{\Omega}^{'}_{ii}(\varepsilon) + \sum_{i<j}^{Nm} \mu_i(\xi(k))\mu_j(\xi(k))[\tilde{\Omega}^{'}_{ij}(\varepsilon) + \tilde{\Omega}^{'}_{ji}(\varepsilon)] < 0 \quad (39)$$

where $\tilde{\Omega}^{'}_{ij}(\varepsilon)$ is given by equation (22).

It is easy to know that inequalities (40) and (41) are sufficient to inequality (39)

$$\tilde{\Omega}^{'}_{ii}(\varepsilon) < 0, \ \forall \varepsilon \in [0, \varepsilon^*] \quad (40)$$

and

$$\tilde{\Omega}^{'}_{ij}(\varepsilon) + \tilde{\Omega}^{'}_{ji}(\varepsilon) < 0, \ \forall \varepsilon \in [0, \varepsilon^*] \quad (41)$$

By using Lemma 2, conditions (40) and (41) are satisfied if LMIs (17),(18), (19) and (20) are feasible.
where R, W and M satisfying the inequality (29).
Multiplying (29) on the left and on the right by $diag\{P^{-1}, P^{-1}\}$, we get:

$$\begin{bmatrix} \tilde{R} & \tilde{W} \\ \tilde{W}^T & \tilde{M} \end{bmatrix} \geq 0 \quad (42)$$

Therefore, the closed loop system is asymptotically stable $\forall \varepsilon \in [0, \varepsilon^*]$, if LMIs (17),(18), (19), (20) and (21) are feasible. This completes the proof.

IV. Numerical Example

The considered discrete nonlinear singularly perturbed system with time delay is:

$$x(k+1) = \begin{bmatrix} 0.7 & 0.3x_2(k) \\ 0.8\varepsilon & \varepsilon \end{bmatrix} x(k) + \begin{bmatrix} 0.12 & 0.42 \\ \varepsilon sin(x_1(k)) & 0.37\varepsilon \end{bmatrix} x(k-d) + \begin{bmatrix} 1 \\ 0.9\varepsilon \end{bmatrix} u(k) \quad (43)$$

with $x(k) = \begin{bmatrix} x_1(k) & x_2(k) \end{bmatrix}^T$
Assume that $\mid x_2(k) \mid \leq 2$.
Non-constant terms are $x_2(k)$ and $sin(x_1(k))$.
By using the CPT, we get a multimodel composed of $2^2 = 4$ sub-models.
According to Lemma 1, $x_2(k)$ and $sin(x_1(k))$ can be written respectively as follow:

$$x_2(k) = F_{1,1}(x_2(k)).2 + F_{1,2}(x_2(k)).(-2) \quad (44)$$

$$sin(x_1(k)) = F_{2,1}(x_1(k)).1 + F_{2,2}(x_1(k)).(-1) \quad (45)$$

with
$F_{1,1}(x_2(k)) = 0.25(x_2(k) + 2)$
$F_{1,2}(x_2(k)) = 0.25(2 - x_2(k))$
$F_{2,1}(x_1(k)) = 0.5(sin(x_1(k)) + 1)$
$F_{2,2}(x_1(k)) = 0.5(1 - sin(x_1(k)))$
Considering the expression of $x_2(k)$, it can written as follow:

$$\begin{bmatrix} 0.7 & 0.3x_2(k) \\ 0.8\varepsilon & \varepsilon \end{bmatrix} = F_{1,1}(x_2(k)) \begin{bmatrix} 0.7 & 0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix} + F_{1,2}(x_2(k)) \begin{bmatrix} 0.7 & -0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix} \quad (46)$$

In order to make the partition functions appear $F_{2,1}(x_1(k))$ and $F_{2,2}(x_1(k))$, we multipy Eqn. (46), with the sum of these two function which is equal to 1. So, we get the following expression:

$$\begin{bmatrix} 0.7 & 0.3x_2(k) \\ 0.8\varepsilon & \varepsilon \end{bmatrix} = F_{1,1}(x_2(k))F_{2,1}(x_1(k)) \begin{bmatrix} 0.7 & 0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix} + F_{1,1}(x_2(k))F_{2,2}(x_1(k)) \begin{bmatrix} 0.7 & 0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix} + F_{1,2}(x_2(k))F_{2,1}(x_1(k)) \begin{bmatrix} 0.7 & -0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix} + F_{1,2}(x_2(k))F_{2,1}(x_1(k)) \begin{bmatrix} 0.7 & -0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix} \quad (47)$$

$\begin{bmatrix} 0.12 & 0.42 \\ \varepsilon sin(x_1(k)) & 0.37\varepsilon \end{bmatrix}$ can be found in the same way.
Then, we get the following coupled multimodel:

$$x(k+1) = \sum_{i=1}^{4} \mu_i(\xi(k)) \left[A_i(\varepsilon)x(k) + D_i(\varepsilon)x(k-d) + B_i(\varepsilon)u(k)\right] \quad (48)$$

with

$$A_1(\varepsilon) = A_3(\varepsilon) = \begin{bmatrix} 0.7 & -0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix}$$

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$$A_2(\varepsilon) = A_4(\varepsilon) = \begin{bmatrix} 0.7 & -0.6 \\ 0.8\varepsilon & \varepsilon \end{bmatrix}$$

$$D_1(\varepsilon) = D_2(\varepsilon) = \begin{bmatrix} 0.12 & 0.42 \\ \varepsilon & 0.37\varepsilon \end{bmatrix}$$

$$D_3(\varepsilon) = D_4(\varepsilon) = \begin{bmatrix} 0.12 & 0.42 \\ -\varepsilon & 0.37\varepsilon \end{bmatrix}$$

$$B_1(\varepsilon) = B_2(\varepsilon) = B_3(\varepsilon) = B_4(\varepsilon) = \begin{bmatrix} 1 & 0.9\varepsilon \end{bmatrix}^T$$

$$\mu_1(k) = 0.125[x_2(k) + 2][sin(x_1(k)) + 1]$$
$$\mu_2(k) = 0.125[2 - x_2(k)][sin(x_1(k)) + 1]$$
$$\mu_3(k) = 0.125[x_2(k) + 2][1 - sin(x_1(k))]$$
$$\mu_4(k) = 0.125[2 - x_2(k)][1 - sin(x_1(k))]$$

By solving the LMIs of theorem 1 with $\varepsilon = 0.4215$ and $d = 1$, we get:

$X = \begin{bmatrix} 10.2012 & -1.5983 \\ -1.5983 & 11.4832 \end{bmatrix}$, $\tilde{Q} = \begin{bmatrix} 5.9256 & -0.8860 \\ -0.8860 & 6.0323 \end{bmatrix}$, $\tilde{M} = \begin{bmatrix} 0.5210 & -0.3113 \\ -0.3113 & 4.9017 \end{bmatrix}$
$\tilde{R} = \begin{bmatrix} 0.1821 & -0.0700 \\ -0.0700 & 1.4834 \end{bmatrix}$, $\tilde{W} = \begin{bmatrix} -0.3052 & 0.0996 \\ 0.1944 & -2.6944 \end{bmatrix}$, $\tilde{K}_1 = \begin{bmatrix} -4.4144 & -7.2908 \end{bmatrix}$
$\tilde{K}_2 = \begin{bmatrix} -5.9895 & 4.8588 \end{bmatrix}$, $\tilde{K}_3 = \begin{bmatrix} -3.8300 & -7.2115 \end{bmatrix}$,
$\tilde{K}_4 = \begin{bmatrix} -5.9047 & 4.8004 \end{bmatrix}$

Then

$K_1 = \begin{bmatrix} -0.5441 & -0.7106 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.5325 & 0.3490 \end{bmatrix}$,
$K_3 = \begin{bmatrix} -0.4844 & -0.6954 \end{bmatrix}$
$K_4 = \begin{bmatrix} -0.5248 & 0.3450 \end{bmatrix}$

Then, the closed loop system is asymptotically stable $\forall\varepsilon \in [0, 0.4215]$ and $d \in]0, 1]$ as shown in figure 2.

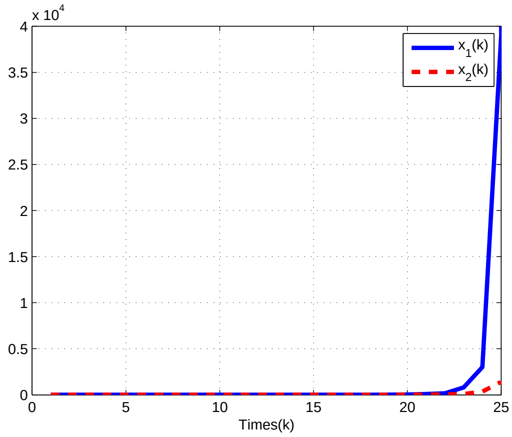

Fig. 1. Evolution des tats en boucle ouverte avec $\varepsilon = 0.4215$ et un retard $d = 1$

From the simulation results, it can be seen that the controller has been able to improve the bound stability of the closed-loop system.

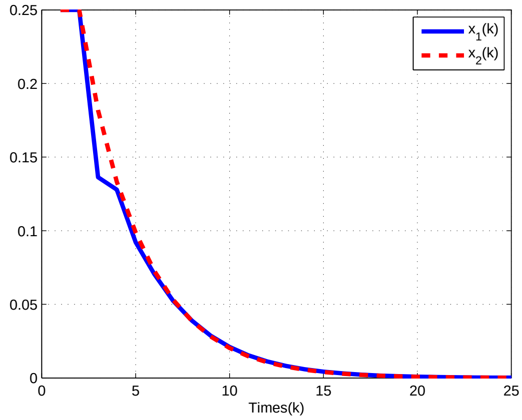

Fig. 2. Evolution des tats en boucle ferme avec $\varepsilon = 0.4215$ et un retard $d = 1$

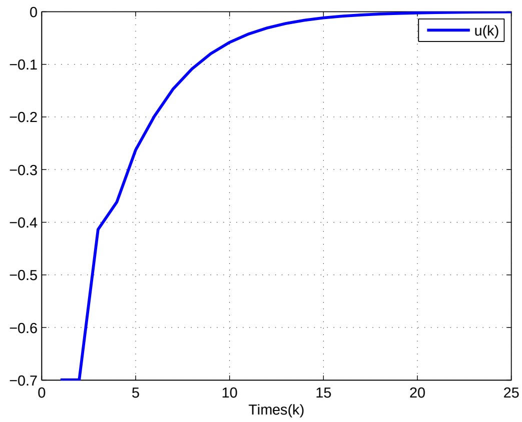

Fig. 3. Evolution de la commande du systme avec $\varepsilon = 0.4215$ et un retard $d = 1$

V. Conclusion

State feedback stabilization is proposed for discrete nonlinear SPS with time-delay. The considered system is represented by a discrete coupled multimodel. Sufficient conditions for the existence of state feedback controller are obtained based on an appropriate Lyapunov function. Simulation example is given to illustrate the effectiveness of the presented method.

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